Lecture 4

CSE-4101

First Order Predicate Logic

So far, we have seen propositional logic can able to control some meaningful information from existing information.

Sometimes the propositional logic cannot control anything which is meaningful, but our brain can control. For example,

Let us consider this information.

***If it snows today, then we will go skiing***

***It is snowing today***

***∴We will go skiing***

This is an argument that propositional logic can handle easily.

Let p denotes “***it is snowing today***”

Q denotes “***We will go skiing****”*

Then we can represent the sentence “***If it snows today, then we will go skiing”*** as p→q, “***It is snowing today***” as p and “***We will go skiing***” as q.

Hence, the information can be written as,

p→q

p

∴q

From the set of premise p→q and p, we can conclude q easily by the rules of modus ponens. So, the set of premises is true then the conclusion is true.

Let us discuss the argument which propositional logic cannot handle.

***Everyone enrolled in the university has lived in a dormitory,***

***Mia has never lived in a dormitory.***

∴***Mia is not enrolled in the university.***

Propositional logic cannot handle this kind of arguments. The first premises “***Everyone enrolled in the university has lived in a dormitory***” itself is a proposition. We cannot break this down it into two propositions. Also, it is a fact, so we cannot break this sentence into two propositions ***p*** and ***q*** and make it as p→q, p ∨ q, p∧q. We cannot break this down. The other sentence is also a proposition. We cannot break it down as well. This is also declaring a fact. “Mia is not enrolled in the university.” is also a proposition. Although, we think logically that this argument seems correct, but we need to prove the validity. Let say the first proposition is denoted by ***p***, and the second proposition is ***q***, and the concluding proposition as ***r***. Hence the argument can be written as

p

q

∴r

This does not make any sense. Propositional logic could not handle these types of argument. As no rules of propositional logic allows us the truth of r, Therefore, we need a more powerful type of logic called first order logic or predicate logic.

So, what is the problem with propositional logic is that the statement “***Everyone enrolled in the university has lived in a dormitory***” has information about Mia but we are not extracting it. We are treating the entire thing as one proposition P, we are not going inside P. When you don’t go inside P, you cannot understand the real meaning inside it.

But First Order Logic Does it. That’s why First order logic is sophisticated and better than propositional logic. For first order logic uses the idea is called subject and predicate. Every statement in First Order Logic has two parts: one is subject and second one is predicate.

To explain subject and predicate, we take a statement “***x is an integer***” In this statement ***x***is a subject. Then we are giving a property to ***x*** that ***x*** is an integer. This property is called a predicate.

Let us consider another example: “Pinky is a cat”. Since we are taking about Pinky, Pinky is a subject. Being a cat is a property or a predicate.

So, in First Order Logic we identify subject and predicate and we write in short hand notation. So, we need to know what is the advantage of writing in shorthand notation. So, the short hand notation of “Pinky is a cat” is as follows:

First of all, we use predicate and we use it with a symbol say “Cat” Predicate.

So, **Cat(x)** (Cat of x) is nothing but “x is a cat”.

That is **Cat(x)**=x is a cat.

Then. **Cat (Pinky)**= Pinky is a cat.

If we have a statement that “Bunny is a cat” then still we can convert it to **Cat (Bunny)**.

That is the idea of writing in short hand notation.

Once we write it, we can repeat it for so many statements.

Similarly for the statement “***x is an integer***”,

We can write it as **Int(x)=x** is an integer.

If we have a new statement that “***y is an integer***”,

We can write it as **Int(y)=y** is an integer.

That is the started point of First Order Logic.

Now we will go through some complex sentences.

* Some cats are intelligent
* Every lion drink coffee

To represent the sentences, we have to use the variables, *x*, *y*, *z* etc.

* Let us consider the sentence “Every lion drink coffee”

Now, whenever we are using the variable ***x,*** we have to identify what it refers. ***x*** may be coffee, drinking or lion anything. ***x*** is nothing, but matter of discussion. We have to fix it.

Figure: Domain of the Discussion or Universe of Discourse

Example of domain of discussion is Lions.

Lions

Then it refers that x is one of the lions. That is the relationship of *x* and universe of discourse or domain. Once, we have defined the Universe of Discourse, now we have to find how to convert it into symbolic logic. Assume that there are three lions *x1*, *x2*, *x3*. There is particular element, that is *x1* is one lion, *x2* is another lion and so on.

Now “Every lion drink coffee” means- “x1 drinks coffee ∧ x2 drinks coffee ∧ x3 drinks coffee”.

Now we will convert it to predicate logic as

coffee(x1) ∧ coffee(x2) ∧ coffee(x3)

However, if there are infinite lions then it will be

coffee(x1) ∧ coffee(x2) ∧ coffee(x3) ……….

Sometimes we do not know how many objects are there in Universe of Discourse. So, we need a short hand notation. For this, we use universal quantifier denoted as ∀x, meaning “every lion has a property of x”. It can be written in predicate notation as

∀x, coffee(x)

Let us consider another example: “**Some cats are intelligent**”

Let us consider that the example of domain of discussion is Cat. Hence the Universe of Discourse is Cats. Let us consider three cats C1, C2 and C3. If we convert it into predicate logic, it will be: C1 is intelligent ∨ C2 is intelligent ∨ C3 is intelligent.

Now we will convert it to predicate logic as

intelligent(x1) ∨ intelligent(x2) ∨ intelligent(x3)

However, there are not infinite Cat.

Now, we will convert it to predicate logic using Existential Quantification.

∃x, intelligent(x) (∃x, there is at least one cat)

Let us consider that the of domain of discussion is animal. Suppose there are three animals a1, a2 and a3. Now we have to written as

Animal

**If a1 is a lion then it drinks coffee**

**If a2 is a lion then it drinks coffee**

**If a3 is a lion then it drinks coffee**

Then we convert it to predicate logic:

Here “a1 is Lion: is one proposition and “it drinks coffee” is another proposition.

Then in predicate logic in can be written as:

**Lion (a1)→coffee(a1)**

Where Lion(a1) means a1 is a Lion.

If we generalize it, we can write Lion (*x*) =*x* is a Lion

Similarly, coffee(*x*): *x* drinks coffee.

We know,

**Lion (a2) →coffee(a2)**

We can write it as

**Lion (a1) →coffee(a1)**

**∧**

**Lion (a2)→coffee(a2)**

**∧**

**Lion (a3) ∧ coffee(a3)**

The combination of these statements is **“*Every Lion drink coffee*”**

It can be observed that there is a commonality between the above three components.

The template **“Lion ( ) ∧ coffee( )”,** repeated three times**,** because there are three lions in the domain. Hence it is true for all lions in the domain.

So, in short hand notation, it can be written as,

**∀x [Lion(x)→Coffee(x)]**

Let’s consider the second statement **“Some cats are intelligent”.** Again, Universe of Discourse is animal**.**

Animal

Here the previous solution will not work. In previous solution we have written as,

**∃x, intelligent(x) (means some animals are intelligent)**

Since the domain has been changed from cats to Animal, this formula will not work, since we want to write the statement that states “Some cats are intelligent”. Let us represents some animal a1, a2 and a3.

a1

a2

a3

ANIMAL

In predicate logic it can be written as,

**∃x[Cat(x) ∧ Intelligent (x***)*]*,* means, there. exist at least one animal, which satisfy the property inside the bracket. That means “There should be an animal which is a cat and the cat is Intelligent.”

So, it can be seen that whenever the domain is changed still, we can answer the question.

∀ and ∃ are building blocks for writing any predicate logic.

We cannot write the statement as, ∃x [Cat(x)→I(x)].

If this predicate is TRUE, then it will give truth value for every interpretation. Let takes three animals, a1, a2, a3, in the universe of discourse.

Intelligent

|  |  |  |
| --- | --- | --- |
| a1  a2  a3 | Cat  Cat  Dog | X  X  √ |

If we want to find out the truth value of the statement, we have to write the formula as,

[Cat(a1)→I(a1)]∨[Cat(a2)→I(a2)]∨[Dog(a3)→I(A3)]

The last statement is FALSE, since we are replacing Cat with Dog.

However, the overall formula is true, since they are ORed together. Hence the truth value of the formula is TRUE. However. The truth value of ∃x [Cat(x)→I(x)] is false as shown from the interpretation. That is why “some cats are intelligent” is different from “there exist some cats implies cats are intelligent”. For the next one there must be at least one cat who is intelligent which will result in TRUE.

So, the short-hand notation ∃x [Cat(x)→I(x)] is not representing some cats are intelligent. But, **∃**x[Cat(x) ∧ Intelligent (x*)*] represents some cats are intelligent.

Let take some Example: “Every student in this class has visited Africa or America”. To convert it into predicate logic, let takes a domain of people.

People

When we take domain as person that the person might be a student, otherwise might not be a student. We need a property to differentiate the students from people. So, we can use a property called “student(x): x is a student in this class”. Since the domain is bigger than students, so we need a function called student.

Now, let vaf(x): x has visited Africa. Also, vam(x): x has visited America.

∀x[student(x)→vaf(x)∨vam(x)]

Let us consider another example: “Some prime number is even number”.

Numbers

We need one property called prime number.

Prime(x): x is prime number.

There is another property called being even number.

Even(x): x is even number

So, in predicate logic, it can be written as:

∃x[Prime(x)∧Even(x)]

2 Variable Predicates: Two variable predicates involves two variables.

For example: Likes(x, y): x likes y

Married(x, y): x married y

Brother(x,y): x is brother of y

Let see, how to represent “Raju Likes Rani”. It can be written as

Likes(Raju, Rani).

Now let see, how to write” Raju Likes Everyone”.

People

x1

x2

x3

It can be written as Raju likes x1 ∧ Raju likes x2 ∧Raju likes x3

If we put it in formula; It will be

likes(Raju, x1)∧Likes (Raju, x2)∧likes (Raju, x3)

In predicate logic it can be written as: Likes(Raju, x) (??)

Let see how to represent “Everyone likes everyone”

Raju likes everyone ∧ Rani likes everyone ∧ ……..

Let’s Take a domain,

.

Raju

Rani

Vani

Now convert it to first order logic.

∀(x) Likes (Raju, x) ∧ ∀(x) Likes(Rani, x) ∧∀(x) Likes(Vani, x)

The ultimate predicate form is,

∀(y) [∀(x) Likes (y, x)], here a different variable is taken because already we have already used y for another intension.

Some other examples of predicate logic are:

1. Some one likes some one

∃x ∃y Likes(x,y)

1. Some likes every one

∃y [ ∀x Likes (y, x)]

1. Everyone likes some one

∀y [∃x Likes (y, x)]

1. Everyone is liked by someone.

∀x[∃y Likes (y, x)]

1. Someone is liked by everyone.

∃y[∀x Likes (x, y)]

1. Nobody likes everyone.

∀y [! ∀x Likes (y, x)]

Example

1. None likes smug people.

1. No mountain is higher than itself.

1. All students get good grades if they study.

1. Every person knows at least one fact.

1. The only good extraterrestrial is a drunk extraterrestrial.

1. Everyone loves everyone else.

1. Every student except George smiles.

1. Every boy who loves Mary hates every boy who Mary loves.

1. Every student who walks talks.

1. All happy employees are not hard working.

Exercise:

1. No cat has a tail.
2. There is exactly one student that received a grade A in CSC 411.
3. Only one CS class is named “Artificial Intelligence.
4. All red apples are delicious.
5. Some apples are not red, yet they are delicious.
6. The Barber of Seville shaves all men who do not shave themselves.
7. All students get good grades if they study.
8. Every person knows every fact.
9. There is a person who knows at least one fact.
10. No person knows every fact.

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